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**Assignment 4**

A thin slab is loaded by distributed force on its outer edge as shown in the figure. Determine the vertical displacement of the outer edge 2-3 by using a bi-linear interpolation to the nodal values. Edge 1-4 is welded to a rigid wall so that the displacements vanish. Thickness of the slab *t*, Young’s modulus *E*, and Poisson’s ratio  are constants. Assume plane stress conditions. Simplify the setting with conditions  and .

*x,X*

*y,Y*

*f*

*L*

*L*

1

1

2

3

4

**Solution template**

Under the plane stress conditions, the virtual work densities (virtual works per unit area) of the thin slab model are given by

 and where

.

Expressions take into account the internal forces (stress) and the external area forces acting on the element domain. The external forces  and  (tractions per unit length in this case) acting on the element edges can be taken into account by a separate force element with the density expression (per unit length)

.

The approximation on the boundary is just the restriction of the element approximation to the boundary (corresponds to a linear two-node element).

Only the shape functions associated with nodes 2 and 3 are needed as the other nodes are fixed (displacement vanishes). By deducing the expression, i.e., combining the linear shape functions in the directions and directions

 and .

In terms of the vertical displacement component  of node 2, approximations to the displacement components and their derivatives are

   and ,

   and .

Virtual work density of the internal forces simplifies to (when the approximations are substituted there)

.

Virtual work density is constant in this case. Integration over the element gives the virtual work expression of internal forces

.

Virtual work expression of external distributed force components  and  is obtained as an integral over the edge defined by . The restriction of approximation to  is given by

 and 

so the virtual work density expression simplifies to



giving the virtual work expression

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Virtual work expression is the sum of internal and external parts

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Principle of virtual work and the fundamental lemma of variation calculus in the form  give

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